THE ASTROPHYSICAL JOURNAL, 251:630–638, 1981 December 15

A SEARCH FOR APSIDAL MOTION IN 4U0115+63[1]

R. L. KELLEY, S. RAPPAPORT,[2] M. J. BRODHEIM, AND L. COMINSKY
Department of Physics and Center for Space Research, Massachusetts Institute of Technology

AND

R. STOTHERS
Institute for Space Studies, NASA Goddard Space Flight Center
Received 1981 February 2; accepted 1981 June 1

ABSTRACT

We have carried out a pulse arrival-time analysis of the archival *Uhuru* data from the 1971 transient outburst of the binary X-ray pulsar 4U0115+63. The 3.6 s X-ray pulsations are clearly present in the data, and we show that the average fractional rate of change in pulse period over the 7 yr interval 1971–1978 corresponds to $\dot{P}/P=-2\times10^{-6}$ yr^{-1}. This spin-up rate is consistent with an average source luminosity $\sim$20 times less than that observed during its flare state. The pulse arrival times were tracked for $\sim 7^{d}$, and by combining these data with the 1978 *SAS 3* orbital determination, we place a limit on the advance of periastron in the 4U0115+63 system of $\dot{\omega}\lesssim 2\overset{\circ}{.}1$ yr^{-1} (95% confidence). The analysis also yields an improved value for the orbital period, $P_{orb}=24\overset{d}{.}3162$. The constraints on the companion star imposed by the limit on apsidal motion are discussed.

Subject headings: pulsars — stars: individual — X-rays: binaries

I. INTRODUCTION

The tidal and rotational distortion of the stars in a close binary stellar system results in an advance of the longitude of periastron, or "apsidal motion," of an eccentric orbit. Moreover, the rate of apsidal motion is sensitive to the internal mass distributions of the constituent stars. The measurement of apsidal motion thus provides one of the few experimental tests of models of stellar interiors. In order for significant apsidal motion to be observed, however, a relatively close binary system with well-determined orbital parameters is required, and thus far apsidal motion has been reliably measured in only $\sim$14 binaries (Kopal 1965; see Stothers 1974 for more recent references).

Binary X-ray pulsars provide a potentially important application of the apsidal motion test because of their generally close orbits and the precision with which their orbits can often be measured. Moreover, the neutron star acts essentially as a point mass and therefore does not complicate the interpretation of the results. Such measurements can be important for the study of the companions in X-ray binaries, which may have had a significantly different evolutionary history from the classical, non–X-ray binaries.

[1]This work was supported in part by NASA under contract NAS5-24441 and by NBS under grant CST8380.
[2]1980–81 Visiting Fellow, JILA, National Bureau of Standards and University of Colorado.

Of the seven X-ray pulsars for which binary orbits have been determined, three have moderately eccentric orbits: 4U0900−40=Vela X-1 ($e\approx0.09$; Rappaport, Joss, and McClintock 1976; Rappaport, Joss, and Stothers 1980), 4U0115+63 ($e=0.34$; Rappaport *et al.* 1978), and GX301−2=4U1223−62 ($e\approx0.44$; White, Mason, and Stanford 1978; Kelley, Rappaport, and Petre 1980). The orbit of GX301−2 has recently been determined, but not with sufficient precision to permit a measurement of apsidal motion in the near future. Rappaport, Joss, and Stothers (1980) have analyzed data from 4U0900−40 spanning $\sim$3.5 yr and were able to place an upper limit on the advance of the longitude of periastron ($\dot{\omega}\lesssim 3\overset{\circ}{.}8$ yr^{-1}) and a corresponding upper limit on the apsidal motion constant ($\log k \lesssim -2.5$). These limits are found to be consistent with the value of k expected for the B0.5 Ib companion star HD 77581, provided that the effective surface temperature T_e is greater than $\sim$20,000 K. It is anticipated that apsidal motion will be detectable in the 4U0900−40 system in the next decade.

The orbit of the X-ray pulsar 4U0115+63 was determined by Rappaport *et al.* (1978) following the discovery of the 3.6 s X-ray pulsations by Cominsky *et al.* (1978*a*) during the second recorded flare-up of the source. The orbital parameters were determined with sufficient precision to allow for a measurement of apsidal motion if a second observation of the source could be made. Unfortunately, 4U0115+63 has not been observed to be active since its 1978 outburst. We have therefore resorted to an analysis of the archival *Uhuru* data of the first recorded outburst of this source in early 1971. The results of this analysis are combined with the 1978 *SAS 3* observations to search for apsidal motion over the 7 yr interval between the transient events. From this analysis we have (1) measured the intrinsic pulse period in 1971, and thus the net change in pulse period over the subsequent 7 yr interval, (2) determined the time of periastron passage in 1971 which yields an improved value for the orbital period, and (3) placed an upper limit on possible apsidal motion ($\dot{\omega} \lesssim 2\overset{\circ}{.}1$ yr^{-1}; 95% confidence). The implications of this limit for the properties of the companion star are discussed.

II. OBSERVATIONS

The X-ray source 4U0115+63 was observed by *Uhuru* on several occasions between late 1970 December (just after launch) and mid 1971 February, during which time the source was in a high luminosity state. The satellite was operated its nominal mode of spinning about a fixed axis once every 12 m, and recording X-ray data in two sets of proportional counters that point in opposite directions and also normal to the spin axis (Giacconi *et al.* 1971). The two sets of proportional counters are collimated to $0\overset{\circ}{.}52 \times 5\overset{\circ}{.}2$ and $5\overset{\circ}{.}2 \times 5\overset{\circ}{.}2$ (FWHM), and are designated side 1 and side 2, respectively. A given source is detected for ~2 s with side 1 and for ~20 s with side 2 during each scan.

Throughout the ~2 month observing interval, more than 100 scans across 4U0115+63 were obtained. Most of the high quality data, however, were obtained during a one week interval. We have selected only the data from side 2 in order to obtain the greatest possible source exposure. The time resolution was 0.384 s and the energy range 2–10 keV. For 66 of the scans across 4U0115+63 the satellite orientation and source intensity yielded data with sufficient counting statistics to be useful in the analysis. During several of the scans in which the count rate is particularly high, the 3.6 s pulsations are apparent in the raw data. For purposes of the pulse arrival-time analysis, the data were divided into a total of 11 groups with an average of six scans per group; these are listed in Table 1. The number of scans in each group was chosen to maximize both the available counting statistics (for computing the pulse profiles) and the total number of resultant profiles. The total time interval spanned by the data set is $7\overset{d}{.}3$.

III. ANALYSIS AND RESULTS

In order to estimate the pulse period during the *Uhuru* observations we folded all of the data on day 390 of 1970 (=1971 January 25), during which time the source was particularly bright, modulo trial pulse periods in the range $3.59 < P < 3.64$ s and computed χ^2 values with respect to the mean count rate of the folded data. The result of this procedure is shown in Figure 1. The effects of three relevant frequencies in the data are clearly evident. The highest peaks are due to the 3.6 s pulsations and the sidebands that result from the 12 m rotation period of the satellite ($\Delta P = 0.018$ s). The smaller peaks flanking the larger peaks are the sidebands of the 96 m orbital period of the satellite ($\Delta P =$ 0.002 s).

To determine which of the larger peaks corresponds to the true pulse period, we note that even if the source had been in an active state for the entire 7 yr interval (1971–1978), and during this time the pulsar had been spinning up at the rate measured in 1978 (Table 2), the 1971 pulse period would differ from the 1978 value by only 0.8 ms. The central peak in Figure 1 corresponds to a pulse period that is within 0.2 ms of the 1978 value, while the other two large peaks lead to an implausibly large change in pulse period ($\Delta P \approx 18$ ms). Moreover,

TABLE 1

4U0115+63 Pulse Arrival Time Data

Group	Time Interval[a]	Number of Scans	Pulse Arrival Times[b]	Pulse Number[c]
1	386.0795–386.1564	6	386.1149571	0 } $\pm\Delta n = 0, 1, 2$
2	386.1644–386.2329	6	386.1997736	2,028 }
3	390.1544–390.2574	4	390.2067705	97,829
4	390.3168–390.4029	5	390.3658502	101,632
5	390.4538–390.5400	6	390.4977818	104,786
6	390.5996–390.7893	6	390.6837628	109,232
7	391.2452–391.3921	6	391.3194380	124,428
8	392.0649–392.2381	5	392.1677753	144,707
9	392.2465–392.3849	5	392.3370833	148,754
10	392.7901–392.9374	7	392.8712419	161,522
11	393.0578–393.3777	10	393.2261012	170,004

[a] Days of 1970; day 0.0 UT of 1970 = JD 2,440,586.5.

[b] Heliocentric arrival time.

[c] The pulse numbers for groups 1 and 2 relative to those for groups 3–11 are uncertain to within a small integer (see text).

TABLE 2

PARAMETERS FOR THE 4U0115+63 BINARY X-RAY SYSTEM MEASURED DURING THE 1978 OUTBURST[a]

Parameter	Value
$a_x \sin i$	140.13±0.16 lt-sec
P_{orb}	$24^d309 \pm 0^d021$
K_x	133.65±0.20 km s^{-1}
$f(M)$	5.007±0.019 $M_\odot$
e	0.3402±0.0004
ω	47°66±0°17
τ	JD 2,443,540.951±0.006
P^b	3.6145737±0.0000009 s
$\dot{P}/P$	$-(3.2\pm0.8)\times10^{-5}$ yr^{-1}

[a]From Rappaport *et al.* (1978). Quoted uncertainties are approximate single-parameter 95% confidence limits.

[b]The pulse period is referred to an epoch of JD 2,443,521.0.

TABLE 3

THREE-PARAMETER FIT TO PULSE ARRIVAL TIMES FROM 4U0115+63[a]

Nine Arrival Times	
P(s)...........	3.614658±0.000036
τ^b	377.08±0.17
χ_ν^{2c}	1.20
$\dot{P}/P$ (yr^{-1}) ...	$-(3.3\pm1.4)\times10^{-6}$
P_{orb}(d)	24.3148±0.0016
Eleven Arrival Times $\Delta n=0$	
P (s)	3.6146246±0.0000043
τ^b	376.930±0.025
χ_ν^{2c}	1.05
$\dot{P}/P$ (yr^{-1}) ...	$-(2.02\pm0.17)\times10^{-6}$
P_{orb} (d)	24.3162±0.0002

[a]Uncertainties are approximate single parameter 1 σ confidence limits.

[b]Days of 1970; day 0.0 UT of 1970= JD 2,440,586.5.

[c]The number of degrees of freedom, ν, is 6 and 8 for nine and 11 arrival times, respectively.

the Doppler variations due to the pulsar orbit can shift the location of the peak by no more than 2 ms. We therefore conclude that the central high peak in Figure 1 corresponds to the true pulse period.

The data from each of the 11 groups of scans (Table 1) were next folded modulo the apparent pulse period for that group to obtain a pulse profile. The pulse profile with the best counting statistics was selected as the template profile against which the remaining profiles were cross-correlated to obtain the pulse arrival times

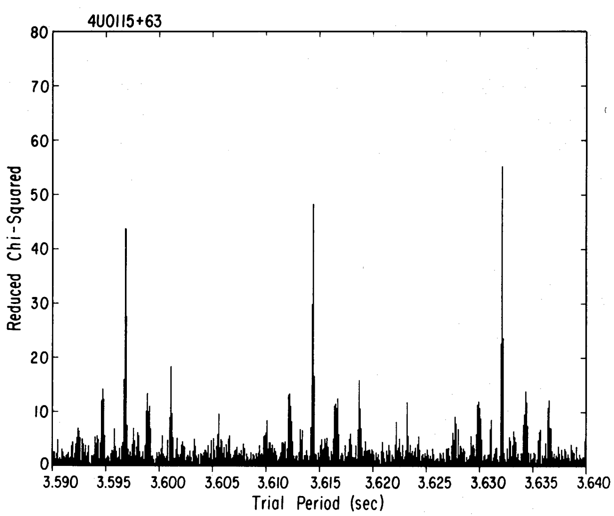

FIG. 1.—Pulsation periodogram for 4U0115+63 produced from data recorded with the *Uhuru* satellite on 1971 January 25. For each trial pulse period the reduced χ^2 value of the folded light curve with respect to a constant (mean) count rate is displayed. The effects of three frequencies are evident in the periodogram. The large peaks are due to the 3.6 s X-ray pulsations with sidelobes that result from the 12 m rotation period of the satellite. The smaller peaks are sidelobes due to the 96 m orbital period of the satellite. The large peak at 3.614 s represents the true pulse period of the source (see text).

(see, e.g., Rappaport, Joss, and McClintock 1976; Rappaport *et al.* 1978). The arrival times were corrected for the Earth's motion about the Sun and are listed along with the corresponding pulse numbers in Table 1. Because of the $\sim4^d$ gap in the data between days 386 and 390, we are unable to determine uniquely the pulse numbers of the two arrival times on day 386 relative to those on days 390–393. In the orbital fits discussed below we have therefore considered various choices for the pulse phasing between days 386 and 390 (indicated by Δn in Table 1).

To determine the orbital parameters for the 1971 data we have carried out χ^2 fits of the pulse arrival times to a function of the form

$$t_n = t_0 + Pn + a_x \sin i F(\theta, e, \omega). \tag{1}$$

Here t_n is the arrival time of the nth pulse, t_0 is a constant, P is the intrinsic pulse period, and $a_x \sin i$ is the projected semimajor axis of the neutron star orbit. The function F describes the Doppler delays in a Keplerian orbit with eccentricity e, longitude of periastron ω, and mean anomaly $\theta = 2\pi(t-\tau)/P_{orb}$, where τ is the time of periastron passage and P_{orb} is the orbital period. We have not included a term involving the change in intrinsic pulse period in equation (1) since the value of $\dot{P}$ determined in 1978 (Table 2) would lead to a cumulative time delay of $\sim$0.2 s over the 7^d3 observing interval, and can therefore be marginally neglected.[3]

[3]We have verified the validity of this approximation by carrying out orbital fits to the data in which $\dot{P}$ was included as a fixed parameter. No significant changes in the orbital parameters were found for assumed spin-up rates up to 10 times the 1978 value (Table 2).

TABLE 4

FOUR-PARAMETER FIT TO PULSE ARRIVAL TIMES FROM 4U0115+63[a]

	NINE ARRIVAL TIMES	ELEVEN ARRIVAL TIMES $\Delta n=1$	ELEVEN ARRIVAL TIMES $\Delta n=0$	ELEVEN ARRIVAL TIMES $\Delta n=-1$
P (s)	3.614678 ± 0.000087	3.614697 ± 0.000012	3.614640 ± 0.000019	3.614590 ± 0.000040
τ^{b}	376.8 ± 1.0	376.65 ± 0.39	377.31 ± 0.40	378.00 ± 0.56
ω (deg)	43.6 ± 15.9	40.7 ± 3.8	51.1 ± 3.6	61.9 ± 4.3
χ_{ν}^{2c}	1.43	1.31	1.06	1.22
$\dot{P}/P$ (yr^{-1})	$-(4.1 \pm 3.5) \times 10^{-6}$	$-(4.89 \pm 0.48) \times 10^{-6}$	$-(2.63 \pm 0.75) \times 10^{-6}$	$-(0.65 \pm 1.59) \times 10^{-6}$
P_{orb} (d)	24.315 ± 0.014	24.3144 ± 0.0044	24.3149 ± 0.0044	24.3153 ± 0.0060
$\dot{\omega}$ (deg yr^{-1}) ...	0.58 ± 2.28	1.00 ± 0.55	-0.49 ± 0.52	-2.04 ± 0.62

[a]Uncertainties are approximate single parameter 1 σ confidence limits.

[b]Days of 1970; day 0.0 UT of 1970 = JD 2,440,586.5.

[c]The number of degrees of freedom, ν, is 5 and 7 for nine and 11 arrival times, respectively.

The values of $a_x \sin i$, P_{orb}, and e were held fixed at the values obtained in 1978 (Table 2) since these quantities are expected to change significantly only on time scales of $\gtrsim 10^6$ yr (see, e.g., Lecar, Wheeler, and McKee 1976; Zahn 1977). For the first set of fits we also held the value for the longitude of periastron fixed at its 1978 value (Table 2). The free parameters in the initial fits were thus t_0, P, and τ. The approximate 1 σ errors assigned to each arrival time were determined empirically to be ~100 ms (see Rappaport, Joss, and McClintock 1976).

Because of the ambiguity in determining the pulse numbers for the arrival times on day 386 relative to those on days 390–393, we first carried out fits to only the nine pulse arrival times during the latter interval. The results of these fits are listed in Table 3. For the fit with nine arrival times, the value of χ_ν^2 is adequately small, indicating that these data do not require a large change in the longitude of periastron between 1971 and 1978. When compared to the 1978 values, the best-fit values of P and τ lead to $\dot{P}/P = -(3.3 \pm 1.4) \times 10^{-6}$ yr^{-1} and $P_{orb} = 24\overset{d}{.}3148 \pm 0\overset{d}{.}0016$ (1 σ uncertainties). The value of P_{orb}, defined here as the difference between times of periastron passage divided by the number of orbital cycles (1971–1978), is in agreement with the value determined in 1978 (Table 2) but is more precisely determined.

When three-parameter (t_0, P, τ) fits were carried out to all 11 arrival times, only the pulse numbers with $\Delta n = 0$ for the two arrival times on day 386 yielded acceptable χ_ν^2 values. The best-fit parameters with $\Delta n = 0$ are also given in Table 3. The net change in the pulse period over the 7 yr interval corresponds to an average spin-up rate of $\dot{P}/P = -(2.0 \pm 0.2) \times 10^{-6}$ yr^{-1}. This is more than an order of magnitude smaller than the value obtained during the 1978 flare state. An improved value for the orbital period, $P_{orb} = 24\overset{d}{.}3162 \pm 0\overset{d}{.}0002$, is also obtained from this fit.

We next included ω as a free parameter and again computed fits to nine and 11 arrival times. In the case of the fit to nine arrival times, we obtain $\omega = 43\overset{\circ}{.}6 \pm 15\overset{\circ}{.}9$ (Table 4) which is consistent with no apsidal motion

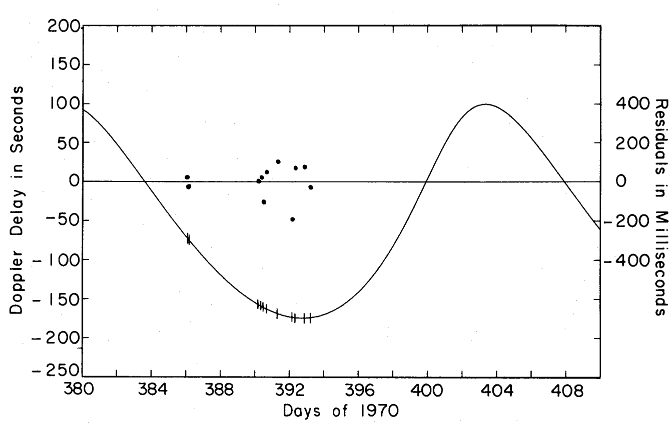

FIG. 2.—Heliocentric pulse arrival-time delays of 4U0115+63 with respect to a constant pulse period. The solid curve corresponds to the best-fit orbital solution with $\Delta n = 0$ (Table 4). The values of $a_x \sin i$, P_{orb}, and e used in the fit are from Table 2 (see text). The errors plotted for each point are ~70 times larger than the actual uncertainties.

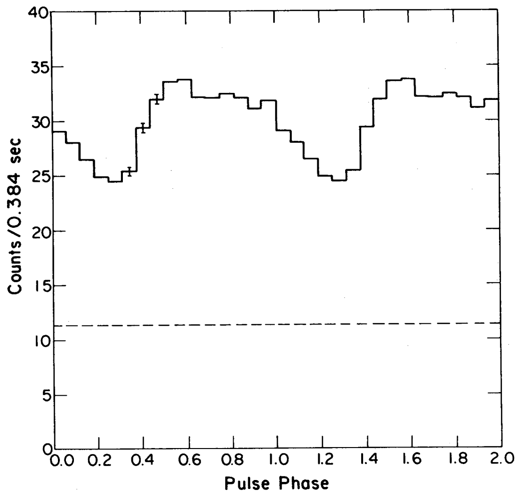

FIG. 3.—The average pulse profile of 4U0115+63 in the 2–10 keV range. The data have been folded modulo the best-fit pulse period for the case where $\Delta n=0$ (Table 4), and with the effect of the binary orbit removed. The dashed line indicates the level of the nonsource background.

over the 7 yr interval ($\dot{\omega}\lesssim 5°$ yr^{-1}, 2 σ). When we include all of the 11 arrival times in the fits, we obtain acceptable values of χ^2_ν for three different assignments of pulse numbers on day 386. For the cases where $\Delta n=1$, 0, and -1, values of $\chi^2_\nu=1.31$, 1.06, and 1.22 were obtained, respectively (Table 4). For the case with $\Delta n=-1$, the inferred value of $\dot{\omega}=-2.^\circ0\pm0.^\circ6$ yr^{-1} implies a significant *decrease* in the longitude of periastron with time, which is opposite in sign to that generally expected for an apsidal motion resulting from a quadrupole interaction[4] (but see Appendix). We therefore tentatively reject the $\Delta n=-1$ solution as physically implausible. The $\Delta n=0$ solution yields a value for $\dot{\omega}$ consistent with zero, $\dot{\omega}\lesssim 0.^\circ6$ yr^{-1} (2 σ), and, as in the three-parameter fits discussed above, yields the best minimum χ^2_ν value. The $\Delta n=1$ solution, however, is also acceptable and yields $\dot{\omega}\lesssim 2.^\circ1$ yr^{-1} (2 σ). Because we cannot distinguish statistically between these latter two solutions, we adopt as an upper limit to apsidal motion $\dot{\omega}\lesssim 2.^\circ1$ yr^{-1} (2 σ). The values of the orbital period given in Table 4 are obtained from the relation $P_{\rm orb}^{-1}=P_{\rm per}^{-1}+P_{\rm aps}^{-1}$, where $P_{\rm per}$ is the time between successive periastron passages and $P_{\rm aps}=2\pi/\dot{\omega}$ is the time for the longitude of periastron to precess through one complete cycle.

In Figure 2 we show the Doppler delays corresponding to the $\Delta n=0$ solution in Table 4 along with the 11 measured pulse arrival times. Also shown are the residuals from the fit.

[4]An advance in the longitude of periastron ($\dot{\omega}>0$) is expected if the axis of rotation of the companion is at least $\sim35°$ out of the orbital plane (see, e.g., Smarr and Blandford 1976).

The mean pulse profile for the 1971 data was obtained by folding all of the data while correcting for the orbital delays (using the $\Delta n=0$ solution, Table 4) and is shown in Figure 3. The pulse profile (2–10 keV) is essentially the same as that obtained by Johnston *et al.* (1978) with the A-3 experiment aboard *HEAO 1*, although there does appear to be a slight enhancement in the X-ray emission at phases 0.7–0.9 (Fig. 3). The pulsed fraction in the 2–10 keV range, as defined by Johnston *et al.* (1978), is $\sim$30%, which is in agreement with the typical 1978 values.

IV. DISCUSSION

The binary companion of 4U0115+63 has been identified as a Be star that may be rapidly rotating and probably lies along or near the main sequence (Johns *et al.* 1978; Hutchings and Crampton 1981). The measured orbital parameters and transient nature of the source (Forman, Jones, and Tananbaum 1976; Cominsky *et al.* 1978*b*; Rappaport *et al.* 1978) indicate that the companion star probably does not fill its critical potential lobe, even at periastron passage. Instead, the identification with a Be star suggests that the mass loss necessary to drive the accretion may occur through the ejection of matter in an equatorial disk or circumstellar shell. Evidence for this type of phenomenon in the pulsing X-ray source GX304−1 has been reported by Parkes, Murdin, and Mason (1980).

Under the assumption that no significant apsidal motion has occurred (Table 3; 11 arrival times), the average fractional change in pulse period over the interval 1971–1978 is $\dot{P}/P=-(2.0\pm0.2)\times10^{-6}$ yr^{-1}. Based on the accretion torque model of pulse period changes (Pringle and Rees 1972; Lamb, Pethick, and Pines 1973; Rappaport and Joss 1977; Ghosh and Lamb 1979), this spin-up rate is consistent with an average source luminosity that is $\sim$20 times less than during the active states (Cominsky *et al.* 1978*b*; Rose *et al.* 1979). A high-sensitivity observation of the source during its quiescent state may be quite useful in testing the predictions of the accretion torque model over a wide range of spin-up rates and X-ray luminosities. The measured value of the orbital period, $P_{\rm orb}=24.^{\rm d}3162\pm0.^{\rm d}0002$, represents an improvement by a factor of $\sim$50 over the 1978 period determination (Table 2). The more accurate period will enable future observations of very brief duration ($<1^{\rm d}$) to yield precise determinations of the intrinsic pulse period because the orbital phase can now be predicted to better than 1° for the next several decades.

The measured limit on $\dot{\omega}$ discussed in § III ($\lesssim 2.^\circ1$ yr^{-1}) can be used to set corresponding limits on the apsidal motion constant of the companion star (Cowling 1938; Sterne 1939). The expression relating the apsidal motion constant to the rate of periastron advance is given by

$$\dot{\omega}=\frac{2\pi k}{P_{\mathrm{orb}}}\left(\frac{R_c}{a}\right)^5\left[15qf(e)+\Omega^2(1+q)g(e)\right], \quad (2)$$

where it has been assumed that the spin angular momentum of the companion star is parallel to the orbital angular momentum. In this expression q is the ratio of the neutron star mass to the companion star mass, R_c is the mean radius of the companion star, a is the full semimajor axis of the orbit (a_x+a_c), and Ω is the ratio of the rotational angular velocity of the companion to the orbital frequency $(2\pi/P_{\mathrm{orb}})$. The quantities $f(e)$ and $g(e)$ are dimensionless functions of the orbital eccentricity and are given by

$$f(e)=\left(1+\frac{3}{2}e^2+\frac{1}{8}e^4\right)(1-e^2)^{-5} \quad (3)$$

and

$$g(e)=(1-e^2)^{-2}.$$

The apsidal motion constant, k, represents a one-parameter description of the internal matter density distribution of the companion star (see, e.g., Schwarzschild 1958). (The contribution to apsidal motion in the 4U0900−40 system from the rotational distortion of the primary, HD 77581, was neglected in Rappaport, Joss, and Stothers 1980. However, this omission did not affect any of their results or conclusions.)

For the 4U0115+63 system, the parameters of the companion star are not sufficiently well known to allow a unique limit on k to be set. We have therefore computed contours of constant upper limits on k, $k_{\max}$, in the mass-radius plane for various choices of the rotation parameter Ω. To compute these contours of $k_{\max}$, we assume a typical neutron star mass of 1.4 $M_\odot$ (see, e.g., Rappaport and Joss 1981) and use the measured values of $a_x \sin i$, P_{orb}, and e (Table 2). The full semimajor axis a of the binary system can be expressed as

$$a=a_x \sin i(1+q)/\sin i, \quad (4)$$

and for each choice of M_c a value for the orbital inclination angle can be determined from the measured mass function (Table 2):

$$\sin^3 i=f(M)(1+q)^2/M_c. \quad (5)$$

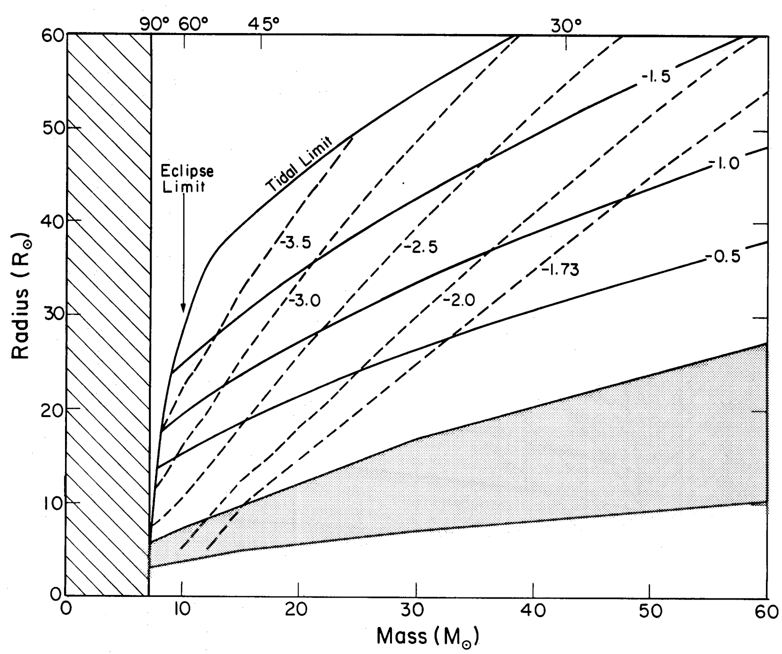

FIG. 4.—Upper limits (*solid contours*) on the apsidal motion constant, $k_{\max}$, plotted in the mass-radius plane of the companion for the 4U0115+63 system. The limits are deduced from the measured upper limit on apsidal motion ($\lesssim 2^\circ\!.1$ yr^{-1}) and the assumption of a nonrotating companion (see eqs. [2−5]). Each contour is labeled with the value of log $k_{\max}$. The lower mass limit at ~7 $M_\odot$ is obtained from the measured mass function (Table 2) and the assumption of a neutron star mass equal to 1.4 $M_\odot$. Several representative values of the orbital inclination angle are indicated along the upper abscissa; these have a unique correspondence with the values of stellar mass shown along the bottom axis. The eclipse and tidal limits represent the maximum radii of the companion star before eclipses would be observed (for a spherical companion), or before the star would overfill its tidal lobe, respectively. The dashed contours are calculated values of log k for stars with T_e=20,000 K (see text). There are no realistic stellar models with log $k \gtrsim -1.73$. For lower values of T_e the contours of calculated values of k shift to the left. The approximate range of masses and radii for standard main-sequence stars is indicated by the shaded region.

By combining equations (2)–(5) we obtain the contours of constant log k_{max} shown in Figure 4 (solid curves) for the case $\Omega=0$.

For a fixed limiting value of $\dot{\omega}$, the limit on k becomes more restrictive with increasing Ω (eq. [2]). To obtain the strongest possible limit on k, we have computed the contours of k_{max} under the assumption that for each mass and radius, the companion star is rotating at the maximum rate, $\Omega=\Omega_{max}$, consistent with not overfilling its critical potential lobe at periastron passage. The functional form of the effective potential in an eccentric orbit is given by Avni (1976). The effective radius of the critical potential lobe is taken to be the radius of a sphere whose volume is equal to that of the critical potential lobe. We note that since the companion may be a rapidly rotating Be star, the above assumption may not be too unrealistic (see, e.g., Slettebak 1979; Hutchings, Nemec, and Cassidy 1979). The values of log k_{max} computed under the assumption of "rapid" rotation are shown in Figure 5 (solid curves). Values of Ω_{max} in Figure 5 range from 0 along the "tidal limit" curve to ~8 and ~50 roughly along the curves for log $k_{max}=-2.0$ and -1.0, respectively.

Also shown in Figures 4 and 5 are the contours of log k (dashed curves) calculated for a range of stellar models with a representative surface temperature, T_e, of 20,000 K (Hutchings and Crampton 1981; see Stothers 1974 and Rappaport, Joss, and Stothers 1980 for a description of the methods and assumptions used in computing the apsidal motion constants). For the case where $\Omega=0$ (Fig. 4), the measured limits on k are seen to be 1–2 orders of magnitude larger than the computed values, so that no apsidal motion should have been detected and no useful constraints on the mass and radius of the companion star can be set. For example, a typical value of $\dot{\omega}$ expected for an early-type main-sequence companion ($M\approx 15\ M_\odot$, $R\approx 10\ R_\odot$) is ~0'.3 yr^{-1}, whereas our limit on $\dot{\omega}$ is 2°.1 yr^{-1}.

For the case with $\Omega=\Omega_{max}$ (Fig. 5), the effects of uniform stellar rotation on the calculated values of k were taken into account using the technique described by Stothers (1974). In this case of "rapid" rotation the measured limits on k_{max} are of the same order of magnitude as the calculated values. In particular, note that companion stars that lie to the right of the hatched line (Fig. 5) have measured upper limits on k that are smaller than the computed values, and can therefore be excluded (for T_e=20,000 K). Similar calculations for T_e= 15,000 and 25,000 K show that the hatched line moves to lower and higher masses, respectively, by ~10 $M_\odot$.

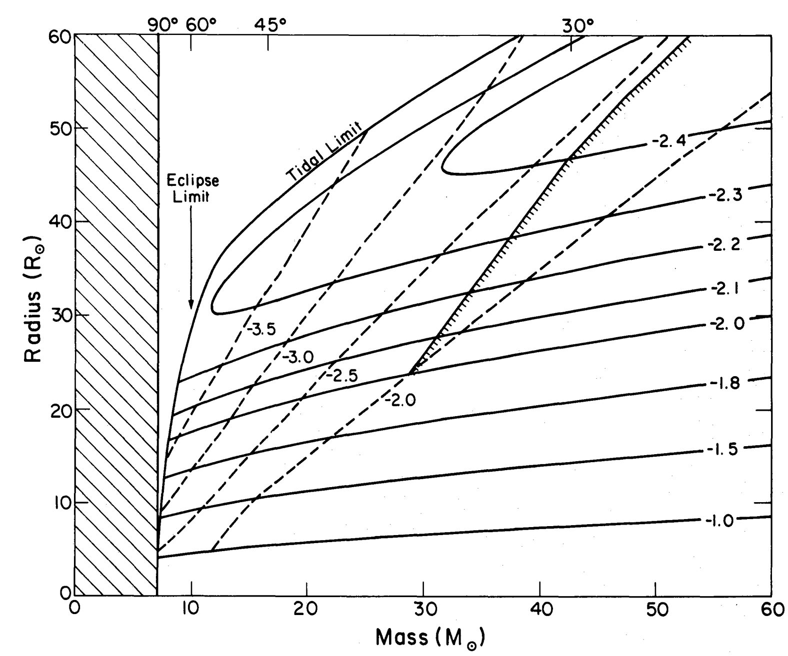

FIG. 5.—Upper limits (*solid contours*) on the apsidal motion constant, k_{max}, plotted in the mass-radius plane of the companion for the 4U0115+63 system. The limits are deduced from the measured upper limit on apsidal motion ($\lesssim$2°.1 yr^{-1}) and the assumption that the companion star is rotating at the maximum rate consistent with it not overfilling its critical potential lobe at periastron passage. The size of the critical potential lobe, R_L, is computed from the expression for the potential given by Avni (1976). The effective stellar radius of the star and R_L are defined as radii of spheres whose volumes equal those of the companion star and critical potential lobe, respectively. The dashed contours are the same as in Fig. 4 except that they have been derived for the case of rapid uniform stellar rotation using the technique described by Stothers (1974). In this case there are no stellar models with log $k\gtrsim-2.0$. For companion stars to the right of the hatched line, the values of k_{max} are smaller than the calculated values of k, and therefore this region of the mass-radius plane is excluded. All other notation is the same as in Fig. 4.

In particular, for $T_e = 15{,}000$ K the range of allowed parameters for the companion star is reduced to a narrow strip in the mass-radius plane.

In summary, we conclude that apsidal motion would have been detectable in the 4U0115+63 system if the companion were a rapidly rotating star with $M_c \gtrsim 30\ M_\odot$ that had evolved off the zero-age main sequence. For smaller masses or slower rotation rates, apsidal motion should not have been detectable. Another orbital determination in the future, with the same precision as the 1978 measurement, should enable us to easily measure apsidal motion if the companion star is rapidly rotating. Apsidal motion will also likely be detectable for most nonrotating companions. Such X-ray observations, coupled with optical observations to measure the rotational velocity of the companion, should lead to a determination of the apsidal motion constant for a star that may be relatively unevolved in an X-ray binary system.

We have recently learned of a new X-ray outburst from 4U0115+63 that occurred in 1980 December and was observed with *Ariel 6* for a total of $\sim 12^{\mathrm{d}}$ (Ricketts and Pounds 1981). It is quite possible that when these new data are analyzed, and the results combined with those presented here, apsidal motion will be detectable.

The authors thank R. Giacconi, W. Forman, and C. Jones of CFA for use of the archival *Uhuru* data, which were obtained under NASA contract NAS5-20048. We are particularly grateful to Fuk Li for valuable contributions during the early stages of this work, and to J. Papaloizou and J. Pringle for sending us a preprint of their paper. We also thank J. B. Hutchings and P. C. Joss for helpful discussions. L. C. thanks Zonta International for support during the course of this work.

APPENDIX

THE EFFECT OF STELLAR OSCILLATIONS ON THE RATE OF APSIDAL MOTION

Papaloizou and Pringle (1980) have suggested that possible resonances between orbital motion and stellar oscillations can seriously influence the rate of apsidal advance in a close binary system (perhaps even reverse the rate). The importance of their effect depends principally on the system having a large value of R_c/a, as in a standard tidal calculation. To obtain effective resonances in which significant amounts of energy are involved, the relevant modes of oscillation must be of quite low order. As Papaloizou and Pringle have demonstrated, the relevant modes will in general be nonradial g-modes or their rotational equivalents.

The 4U0115+63 system is characterized by $0.1 \lesssim R_c/a \lesssim 0.3$ and $0.02 \lesssim q \lesssim 0.20$. The dimensionless quantity

$$\left(\frac{2\pi}{P_{\mathrm{orb}}}\right)^2 \frac{R_c^3}{GM_c} = (1+q)\left(\frac{R_c}{a}\right)^3$$

is thus in the range 0.001–0.03. For a significant resonance to occur, this quantity must be of the same general order as the analogous pulsational quantity, $(2\pi/P_{\mathrm{pul}})^2\, R_c^3/GM_c$. Ignoring the effect of axial rotation on the modal spectrum, we find that an assumed range of 0.001–0.03 corresponds to an extremely high-order g-mode, even for the most favorable case in which a very low degree l of the spherical harmonics is involved in the oscillations and the star has a very low central condensation (see, e.g., Smeyers 1967; Osaki 1975; Stothers 1976). By assuming that quadrupole ($l=2$) oscillations are excited and that the star may be approximated by a polytrope of index $n=3$, we can infer from the work of Vandakurov (1968) on nonradial oscillations of polytropes that the order of the potentially resonant g-modes for the B star in 4U0115+63 must be in the range $37 \lesssim j \lesssim 218$. It is therefore clear that stellar oscillations cannot significantly affect the apsidal motion in this particular binary system, unless the B star is rotating very rapidly, in which case the tidal forcing frequency at any point on the star is very high and possibly in resonance with a very low order g-mode (Papaloizou and Pringle 1981).

For the 4U0900−40 system containing a slowly rotating B0.5 supergiant, Papaloizou and Pringle (1980) have formally demonstrated the existence of an orbital resonance with low-order ($j \approx 3$) pulsational g-modes by assuming a polytrope of index $n=3$ for the stellar model. Although this assumption is acceptable for a zero-age main-sequence star, the B0.5 supergiant in this system has probably evolved close to, or more probably beyond, the end of the main-sequence phase (as Papaloizou and Pringle were aware). The higher central condensation and the denser spectrum of quadrupole pulsational modes for such an evolved star (e.g., Osaki 1975; Stothers 1976) resemble the corresponding properties for a polytrope with an index of at least $n=3.5$ (Robe 1968). It should be pointed out here that there exist large structural differences between polytropes of index $n=3$ and $n=3.5$. Since 4U0900−40 has $(2\pi/P_{\mathrm{orb}})^2\, R_c^3/GM_c \approx 0.23$, the potentially resonant quadrupole g-modes of the supergiant must be at least of order $j \approx 22$ for $P_{\mathrm{pul}} = P_{\mathrm{orb}}$ (Vandakurov 1968). Even for $P_{\mathrm{pul}}/P_{\mathrm{orb}} = 1/3$ (a ratio suggested by Papaloizou and Pringle), the order of the potentially resonant modes would be at least $j \approx 7$ (Robe 1968). It is unlikely that such high modes could have a large effect on the rate of apsidal motion.

It would seem that the resonance effect suggested by Papaloizou and Pringle (1980) may have little influence in the early-type X-ray binary systems that have so far been studied, unless the rotational effects on the g-mode spectrum are unexpectedly large and in the right direction, or unless the toroidal modes (which have not been calculated so far) have important resonances. However, further work based on more realistic stellar models is necessary to decide on this question.

REFERENCES

Avni, Y. 1976, *Ap. J.*, **209**, 574.
Cominsky, L., Clark, G. W., Li, F., Mayer, W., and Rappaport, S. 1978*a*, *Nature*, **273**, 367.
Cominsky, L., Jones, C., Forman, W., and Tananbaum, H. 1978*b*, *Ap. J.*, **224**, 46.
Cowling, T. G. 1938, *M.N.R.A.S.*, **98**, 734.
Forman, W., Jones, C., and Tananbaum, H. 1976, *Ap. J. (Letters)*, **206**, L29.
Ghosh, P., and Lamb, F. K. 1979, *Ap. J.*, **234**, 296.
Giacconi, R., Kellogg, E., Gorenstein, P., Gursky, H., and Tananbaum, H. 1971, *Ap. J. (Letters)*, **165**, L27.
Hutchings, J. B. and Crampton, D. 1981, *Ap. J.*, **247**, 222.
Hutchings, J. B., Nemec, J. M., and Cassidy, J. 1979, *Pub. A.S.P.*, **91**, 313.
Johns, M., Koski, A., Canizares, C., and McClintock, J. 1978, *IAU Circ.*, No. 3171.
Johnston, M., Bradt, H., Doxsey, R., Gursky, H., Schwartz, D., and Schwartz, J. 1978, *Ap. J. (Letters)*, **223**, L71.
Kelley, R., Rappaport, S., and Petre, R. 1980, *Ap. J.*, **238**, 699.
Kopal, Z. 1965, *Adv. Astr. Ap.*, **3**, 89.
Lamb, F. K., Pethick, C. J., and Pines, D. 1973, *Ap. J.*, **184**, 271.
Lecar, M., Wheeler, J. C., and McKee, C. F. 1976, *Ap. J.*, **205**, 556.
Osaki, Y. 1975, *Pub. Astr. Soc. Japan*, **27**, 237.
Papaloizou, J., and Pringle, J. E. 1980, *M.N.R.A.S.*, **193**, 603.
______. 1981, private communication.
Parkes, G. E., Murdin, P. G., and Mason, K. O. 1980, *M.N.R.A.S.*, **190**, 537.
Pringle, J. E., and Rees, M. J. 1972, *Astr. Ap.*, **21**, 1.
Rappaport, S., and Joss, P. C. 1977, *Nature*, **266**, 683.
______. 1981, in *X-Ray Astronomy with the Einstein Satellite*, ed. R. Giacconi (Dordrecht: Reidel), p. 123.
Rappaport, S., Clark, G. W., Cominsky, L., Joss, P. C., and Li, F. 1978, *Ap. J. (Letters)*, **224**, L1.
Rappaport, S., Joss, P. C., and Stothers, R. 1980, *Ap. J.*, **235**, 570.
Rappaport, S., Joss, P. C., and McClintock, J. E. 1976, *Ap. J. (Letters)*, **206**, L103.
Ricketts, M., and Pounds, K. 1981, private communication.
Robe, H. 1968, *Ann. d' Ap.*, **31**, 475.
Rose, L. A., Pravdo, S. H., Kaluzienski, L. J., Marshall, F. E., Holt, S. S., Boldt, E. A., Rothschild, R. E., and Serlemitsos, P. J. 1979, *Ap. J.*, **231**, 919.
Schwarzschild, M. 1958, *Structure and Evolution of the Stars* (Princeton: Princeton University Press).
Slettebak, A. 1979, *Space Sci. Rev.*, **23**, 541.
Smarr, L. L., and Blandford, R. 1976, *Ap. J.*, **207**, 574.
Smeyers, P. 1967, *Bull. Soc. Roy. Sci. Liège*, **36**, 357.
Sterne, T. E. 1939, *M.N.R.A.S.*, **99**, 451.
Stothers, R. 1974, *Ap. J.*, **194**, 651.
______. 1976, *Ap. J.*, **210**, 434.
Vandakurov, Y. V. 1968, *Soviet Astr.—AJ*, **11**, 630.
White, N. E., Mason, K. O., and Sanford, P. W. 1978, *M.N.R.A.S.*, **184**, 67P.
Zahn, J.-P. 1977, *Astr. Ap.*, **57**, 383.

R. L. Kelley, S. Rappaport, M. J. Brodheim, and L. Cominsky: Room 37-551, Massachusetts Institute of Technology, Cambridge, MA 02139

R. Stothers: Institute for Space Studies, Goddard Space Flight Center, NASA, New York, NY 10025